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Short Communication

A CGI Case Study Analyzing Montessori Math Effectiveness

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Abstract

Cognitively Guided Instruction is pushed as a viable method to evaluate student understanding. In a particular Montessori environment, this process revealed much about the thinking processes of students leaving the Montessori program and entering a direct instruction classroom. This paper shows in-depth examples of CGI in practice, both for research purposes and exposure to teachers unfamiliar with the tool. Further, a discussion develops about how students rely on old methods versus learning new methods, including using picture drawings; some conclusions are drawn after some qualitative evidence is given pertaining learning third and fourth grade mathematics topics.

Keywords: Montessori; Mathematics; Social; Manipulatives; Learning Style

1 CGI: What Does It Mean?

Educators have difficulties assessing students' mathematical process and understanding, compared to evaluating correct and incorrect solutions. The recent focus on Cognitively Guided Instruction (CGI) has provided some depth for teachers to peer into the problem solving strategies of their students. CGI is "focused more directly on helping teachers understand children's thinking by helping [teachers] construct models of the development of children's thinking in well-defined content domains" (Carpenter, Fennema, & Franke, 1996, p.5). Teachers request that students learn to avoid memorizing steps, but instead show a process that will lead to better problem solving in situations that are unfamiliar. "To understand, students must get inside these topics; become curious about how everything works; figure out how this topic is the same as, and different from a topic they already studied; and become confident that they would handle problems about the topic, even new problems they have not seen" (Hiebert & Wearne, 2003, p.1). To D'Ambrosio (2003), the whole purpose of teaching mathematics through problem solving is for such understanding—the learning of new material through trial and error based on past knowledge. Students can create the context that makes the most sense based on their current knowledge base. "There may be debate about what mathematical content is most important to teach. But there is growing consensus that whatever students learn, they should learn with understanding" (Hiebert et al, 2000, p.2).

Students do not naturally use or copy a method they do not understand, and for this reason it is important to allow students to work through their own strategy (Carpenter et al., 1996). "CGI teachers have found that students gradually learn to make sense of the context on their own...students learn to look for the mathematical relationships that are a part of the story and use them to get started on a solution" (Carpenter et al, 2015, p.139). Rather, CGI "suggests that students' invented algorithms are constructed through progressive abstraction of their modeling procedures…" and "the manipulation of [models] become objects of reflection. Eventually, the words that students use to describe their manipulation…become the solutions themselves" (Carpenter et al., 1996, p.12-13).

2 Effective CGI Construction

Successful CGI assessments should include open-ended problems. Such problems must allow room to display a variety of mathematical strategies. Open-ended problems are meant to elicit a discussion about the process, instead of focusing on the final answer. "It is important in teaching to use open [ended] problems because they encourage pupils to invent different solutions" (Laine et al, 2014, p.126). Research has shown that students who progress through a curriculum focused on more open-ended problems are able to retain a better conceptualization of what they accomplished, and without sacrificing standardized test scores (Stein et al, 2003). "In the context of classroom teaching one major advantage of using open problems and investigations is that, because there are multiple solutions, they cater for a wide range of mathematical abilities and stages of development in children" (Way, 2005, p.1).

A typical CGI assessment concludes with follow-up questions for student, once they arrive at their final solutions, in order for the interviewer or teacher to obtain more insight into students' understanding; this dialogue also gives students a chance to stand by their answers as correct or incorrect (Carpenter et al., 2015). Teachers should not assume why a student carried out a particular strategy, and instead they can ask follow-up questions (Jacobs & Ambrose, 2008). Teachers can then compare how students perceive the problem in terms of original approach in order to fully interpret student understanding (Charalambous, 2010). “Having a student share more details of her thinking engages the student in articulating, explaining, and justifying her thinking and enables the teacher and other students to understand the strategy the student used” (Carpenter et al., 2015, p.140).

3 Methodology

This qualitative study viewed the mathematical understanding of selected fourth grade students (English-fluent) who were in their first year in a non-Montessori program at an international school in Central America, following prior Montessori education. Table 1 shows the details on this sample of six students taken from a group of 16 fourth graders at the same school with similar backgrounds, all mixed into the same classroom. They were chosen by recommendation of their teachers for a sample of students that represented both middle-performing ability, and also a variety of experience at the particular institution. The significance of these students is their experience with heavy implementation of materials in their math education prior to this academic year, through the Montessori program. *Manipulatives* is the term for these tangible objects that students can use, often in a mathematics context, for creating physical representations of abstract ideas (Cope, 2015). Commonly found in Montessori classrooms, manipulatives are often used in three main stages: concrete, representational, and abstract (Stein & Bovalino, 2001). The goal is for students to eventually transform the objects into mathematical symbols—a gradual shift that still maintains student understanding, and an idea toward problem solving mentioned earlier.

Table 1. CGI Sample List

Name	Grade	Montessori Experience
Felix	Fourth	One year
Lior	Fourth	Entire schooling
Victoria	Fourth	Three years
Romeo	Fourth	Two years (with break)
Valentina	Fourth	Three years
Kai	Fourth	Entire schooling

For the six students selected to carry out the CGI portion of the study, the problems gradually challenged them to provide new strategies and create content knowledge, as the literature suggests is important to obtain the most understanding about student thinking (Goldin, 1997). Additionally, teachers that utilize CGI instruction have discovered that students gradually learn to make sense of the context of a problem on their own; more importantly, students who struggle to get started on word problems learn to identify the mathematical relationships within the problems and use them to their advantage (Carpenter et al., 2015). These students were given twelve such problems in one assessment period, one set of six problem in October and one in March; both aligned with curriculum expectations for that period in the academic year. The first round focused on addition and subtraction, with some multiplication included; the second round emphasized multiplication, division, and usage of fractions. In the following section, three examples from these sets are shown as effectively interpreting student thinking and understanding in various situations—in particular they extent they rely on physical objects to create new mathematics knowledge. Validity of the students’ strategies is indicated, as well as their success in providing a correct solution, by the final column of each results summary table. Some dialogue will also be included, and finally, some conclusions will be discussed regarding student understanding and methods.

4 Examples of Effective CGI

4.1 CGI Example #1

Figure 1. CGI Round 2, Question 3

Robin went to a party where each person ate ____ of a pizza. If ____ people ate pizza, how many pizzas were there in all so that they each got to eat ____ of a pizza and there were no leftover pieces?

Source: Empson, S., Junk, D. & Turner, E. “Formative Mathematics Assessments for Use in Grades K-3.”

The use of fractions allowed students to think in terms of multiplication and/or division. The one in the numerator was familiar, as students said that so far the only fractions covered in class were similar. Students expressed different ways of thinking for this fraction-based problem, particularly those that solved it correctly. For example, Valentina quickly decided to draw a circle to represent each pizza, and then divided it into the appropriate fractional pieces. She then meaningfully used the picture by physically counting each piece that represented each person. Kai also used the concept of material objects, using his fingers to count by three to represent the people each eating ⅓ of a pizza, rather than writing it down. Both of these strategies involved visualization rather than algorithms or formulas to achieve the correct answer. These two students created their own objects to use in a difficult, unfamiliar scenario. Alternatively, Victoria was quick to realize that 24 people, each eating ⅓ of a pizza, could be represented as 24÷3. She stated that this problem required recalling division facts rather than multiplication facts, but when pushed to do the problem again with ⅔ as the fraction instead, she admitted less confidence with anything besides a one in the numerator. However, she proceeded with the number sense to realize the solution should be doubled, and correctly answered 16. Felix and Lior struggled to start on the problem, but persevered for nearly ten minutes with more basic strategies, despite admitted weakness in the topic. The results are shown in Table 2.

Table 2. Round 2, Question 3 Results: Measurement with Fractions

Name	Identified Operation	Method	CGI Strategy	Result
Felix	Subtraction	Repeated subtraction, miscounted in process	Skip counting down	Valid/Incorrect
Lior	Division	Building addition	Skip counting all	Valid/Incorrect
Victoria	Division	Division facts	Memorization	Valid/Correct
Romeo	Multiplication	Algorithmic multiplication	Algorithmic	Valid/Incorrect
Valentina	Division	Draw a picture	Written form of direct modeling for grouping	Valid/Correct
Kai	Addition	Building addition through finger counting	Counting all	Valid/Correct

4.2 CGI Example #2

Figure 2. CGI Round 2, Question 4

Okhee has a snowcone machine. It takes ____ of a cup of ice to make a snowcone. How many snowcones can Okhee make with ____ cups of ice?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

In this problem, multiplication is hidden in the process rather than division, like in the previous problem. Yet three students assumed that the presence of a fraction automatically meant division was necessary to solve the problem. Felix began correctly this time, but erred in his reasoning. He said, "¼ is kind of like a quarter, if it was full of ice then it would be three quarters, and then times 20 [cups]." Because of this, he was short on his final solution, as he did not account for the full 4/4 of the cup of ice. Lior and Victoria incorrectly automatically identified the problem as division of whole numbers simply because fractions were included. These two students then analyzed the problem and strictly took the fractional part of all snow cones, arriving at an incorrect solution that was extremely small and did not make sense in context. Victoria, who had performed strongly in both problem sets to this point, looked at her solution with a somewhat confused expression that showed uncertainty, but moved to the next question. Romeo and Kai each ultimately decided to use multiplication for their solution. Romeo realized this strategy immediately by exclaiming, "that's a lot of snow cones!" Kai took longer, at first saying "I think this is division." As shown in Figure 1, Valentina again preferred to draw a picture, this time cutting each shape into fifths. She did not hesitate or confuse the strategy with the slightly different pizza problem by breaking down each circle, and proceeded with a completely different mindset from before. Whereas before she drew pizzas as necessary until she reached her goal, this time she knew to draw 15 boxes before cutting them into fifths. Her work is shown in Figure 1, and the overall results are shown in Table 3.

Table 3. Round 2, Question 4 Results: Partitive with Fractions

Name	Identified Operation	Method	CGI Strategy	Result
Felix	Multiplication	Algorithmic multiplication	Algorithmic	Valid/Incorrect
Lior	Division	Division facts	Memorization	Invalid/Incorrect
Victoria	Division	Division facts	Memorization	Invalid/Incorrect
Romeo	Multiplication	Algorithmic multiplication	Algorithmic	Valid/Correct
Valentina	Multiplication	Draw a picture	Written form of direct modeling for grouping	Valid/Correct
Kai	Division	Recollection of multiplication facts	Multiplication	Correct

4.3 CGI Example #3

Figure 3. CGI Round 2, Question 6

Jane says that if 6 people are sharing 10 cookies each person gets 1 and 2/3 cookies. John says that each person should get 1 and 4/6 cookies. Who is right? Can they both be right?

Source: Empson, S., Junk, D. & Turner, E. "Formative Mathematics Assessments for Use in Grades K-3."

The results of this last problem confirmed that students had some understanding of what fractions represent. Five of the six students quickly decided that the students could each have one cookie but not quite two. They also identified that the remaining four cookies would be split six ways, and believed John's statement to be correct with 4/6 as a representation of this information. Felix only selected John's statement because it was given in fraction form for him, and said he could not have derived the form himself. The other four students who selected John's statement knew that 4/6 was correct because they worked it out independently of the provided statements. Valentina thought that ten people were unable to share six objects. She struggled for a long time, going back and forth with her thoughts and ideas, and Figure 1 shows why this was the case: she had a very difficult time figuring out how to split apart the images with the given information, particularly because of the non-integer final solutions. "This one is hard to draw," she said after nearly ten minutes of silently drawing and erasing. Kai confessed that division was not something he felt he was good at, and guessed, "I think they're both wrong." While many recognized 4/6 as a possibility, no student recognized that 2/3 was the same as 4/6, which was one of the goals for this type of question. While they realized that fractions represented division, they were not strong in fraction equivalency. While students said that four cookies could be split among six people, discussing two cookies split among three people sounded either unrelated or impossible to them. Students also confirmed they were more familiar with remainders than fraction notation to express leftovers in a division problem. No one correctly identified Jane as also being correct in this situation. In fact, they were actually confident in denouncing her answer as absolutely incorrect. Students were therefore partially, rather than fully, accurate with their final answer, as shown in Table 4.

Table 4. Round 2, Question 6 Results: Measurement Division with Equivalent Fractions

Name	Identified Operation	Method	CGI Strategy	Result
Felix	Division	Confirming fractions given in the problem	Guessing	Invalid/Partially Correct
Lior	Division	Using remainders to form fractions	Measurement division	Valid/Partially Correct
Victoria	Division	Using remainders to form fractions	Measurement division	Valid/Partially Correct
Romeo	Division	Using remainders to form fractions	Measurement division	Valid/Partially Correct
Valentina	Division	Draw a picture	Written form of direct modeling for grouping	Invalid/Incorrect
Kai	Division	Guessing	Guessing	Invalid/Partially Correct

5 Conclusions

Valentina and Kai primarily focused on using objects (drawings, finger counting) to help them figure out problems they were less comfortable with. It is important to note that student comments showed many fourth grade students found the use of manipulatives "frustrating," "annoying," and generally described them as a less efficient way of getting through the curriculum. One student, Romeo, even said, "I don't like using…materials. It takes so much longer than writing on a paper…[but] you had to use them (in Montessori)." In Table 5, the overall results of the full 12 combined questions from both CGI sets are shown.

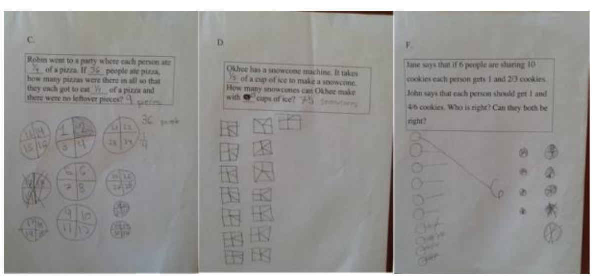
Table 5. CGI Results for Both Rounds

Name	1st Round	2nd Round	Total Score

Felix	4 out of 6	3.5 out of 6	7.5 out of 12
Lior	4 out of 6	2.5 out of 6	6.5 out of 12
Victoria	5 out of 6	4.5 out of 6	9.5 out of 12
Romeo	6 out of 6	4.5 out of 6	10.5 out of 12
Valentina*	5 out of 6	5 out of 6	10 out of 12
Kai*	2 out of 6	5.5 out of 6	7.5 out of 12

Out of the six students tested, four did better on the first round than the second round. The students marked in the table showed a preference toward using objects, materials, or pictures as a visual aid.The two students who did not experience lower scores in the second round were also the students that naturally used visual aids such as drawing pictures or counting objects. While Kai had earlier expressed in interviews that he liked both manipulatives and handwriting equally, many times during the assessments he relied on tapping his pencil, counting his fingers, or thinking in groups—these actions all represented physical objects in his mind. His improved scores may indicate that these skills are helpful in learning new, complicated concepts. Similarly, Valentina had previously described her feelings toward manipulatives as helpful in the past, but also mentioned that using them repetitively became so “boring” and “annoying” that she preferred handwriting in fourth grade. However, Valentina preferred drawing pictures to help with many of the second round division and fraction problems that she admitted were more difficult. Compared to the rest, this was more than any other student in this assessment period. When she felt uncomfortable with a problem, she reverted back to visualization methods, and when she already knew how to approach a problem, she used algorithmic handwriting methods. Valentina particularly struggled when she could not identify a way to illustrate a difficult problem through direct modeling. Figure 1 shows Valentina’s work on the three previously discussed second round questions discussed. Clearly, using manipulatives in their Montessori experience was more essential to their learning than the students may have realized, as the results in Table 5 showed.

Figure 4. Valentina’s Handwritten Work for Round 2 - #3, #4, #6



Overall, students showed more confidence with understanding division than understanding fractions. Students strategized a variety of ways to achieve their goal in division problems, which showed conceptual understanding rather than simply working through an algorithm. Their comments and handwritten work reflected why they were choosing that particular strategy, as per true CGI assessments. Because they covered more unfamiliar topics such as division and fractions, the students who leaned toward manipulatives heavily used them even more during the second round of problems. At the same time, the students who downplayed the importance of manipulatives to their learning struggled more with how to proceed on these new, challenging concepts. Many admitted that manipulatives were helpful at one time, but now they “don’t need them anymore,” even if the assessment showed otherwise. One student, Victoria, said that the content in her new grade may be more difficult, but the style was more helpful (that is, writing down the content instead of using any objects).

Students were more able to identify problems as division in the second round than they were in the first round (earlier in the year). Operations with fractions appeared to be a concept that students could mostly understand as a context for division. Fraction equivalency was present in the final problem and all students not only failed to recognize the equivalency, but also incorrectly believed the proposed equivalent fraction was wrong. Sometimes students thought that the presence of a fraction automatically meant division was the appropriate strategy, and even one of the stronger students fell into this assumption when the problem actually called for multiplication based on the correct interpretation of the situation. These are all findings that were a direct result of these CGI-focused questions, allowing a substantial glimpse into student thought and processing.

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