American Journal of Advanced Research, 2018, 2–1
December 2018, pages 1-6
doi: 10.5281/zenodo.1402812
http://www.ibii-us.org/Journals/AJAR/
ISBN 2472-9264 (Online), 2472-9256 (Print)

Invariant Method in Topological Data Analysis (A Paradigm on Data Shape Approach)

Antonio Alatorre[1,*], F. I. Becerra[2] and J.M. Casillas[2]

[1]Department of Physics, University of Guadalajara,

[2]Department of Mathematics, University of Guadalajara.

*Email: aliagaroth@hotmail.com

Received on March 27, 2018; revised on May 28, 2018; published on August 23, 2018

Abstract

This paper works as a motivation to consider stronger methods in TDA (Topological Data Analysis). We discuss some of the main principles used currently in Big Data analysis. This discussion points became evident while we apply TDA to Big Data. Mathematical formal concepts might be considered, in order, to obtain better results from Data Analysis. Among these arguable principles, it may be considered “data grouping methods”, and also “relation data base-graph and its connection”. We discuss the topological argument, which stands as a powerful tool on Big Data analysis and TDA, from “inverse numbers” methodology, and we present a qualitative description of “Invariant Method” which we think is a strong methodology in order to obtain valuable hidden information from Big Data sets. Furthermore, we propose an algorithm to invariant method.

Keywords: Topological Data Analysis, Big Data, Data Science, Data Shape, Topological Invariants.

1 Introduction

Big Data is one of the most interesting and complex problems to solve not only for science and technology but also for society itself. It demands stronger hardware systems to warehouse greater amounts of data, it demands capability to build adequate software to process not structured and structured data. It also asks for a better user/client interface design. Besides, Big Data philosophy demands an upgrade on timing while queries are required. Big Data is not only a new technology or approach to manage data, it is also a new view of information tool. One can say accurately, that Big Data is (currently) the greater practical information tool developed until now. This fact is evident, and it also forced and invited many other general sciences to contribute in some way. Mathematics and Data Scientific analysis based sometimes in linear algebra and statistical processes, tried in many ways to add methods, point of views and strategy in order to benefit Big Data Analysis. One of the stronger and workable approach from mathematics to data analysis. TDA, which is based on topology and applications to data, and Big Data, has been this select product offered in benefit to extract information, that perhaps using other kind of methods, one would ever obtain [2]. Topology, in general terms, as a mathematical branch, is a complement discipline which involves mathematical analysis of continuity, abstract algebra and mathematical objects as metric spaces, number theory, from where many properties of study objects environment and landscape arise and stands for topological arguments, also, geometrical approach, as the general idea of differentiability over vector fields and manifolds. The direct benefit to data analysis came from topology lies on the possibility to reconstruct an image or model of the real systems given a data set measured over one time-interaction convergence point. This method is called "inverse numbers" [1] and it has been used as a general strategy, not only in Big Data approach, and not only with topology applications, thru other disciplines and information sets.

The main purpose of our research group currently, has been develop of a TDA "invariant method", although in this article, we considered necessary to present an *apriori* vision of this same methodology and the tools that we are considering. Reasons to use certain kind of tools matters. It exists a real difference among geometrical data analysis and TDA. For the first one, the nature of the structure wanted is supported on distance between clusters. It might exist an induced and well define metric, while in TDA what matters is exclusively the shape and its invariant transformations. Therefore, an ontological method debate is worth it. This discussion represents for us the opportunity to present arguments before ordinary methodologies and current tendencies on TDA study. Further we will describe qualitatively our topological invariant method. Technical details and formalism about this method might be presented in an upcoming paper.

1.1 Data Science Roots and its Global Understanding.

In order to improve and acquire new and powerful knowledge and skills as a tech community in the world, we must understand history and deep roots of Data Science. There are two main branches, for which data science stands. First one is scientific data analysis approach. In the practice, these operations consisting in statistical analysis on data after an experiment or a set of experiments. Measuring always have been one of the greatest problems in sciences, but nowadays this problem has evolved to a complex one; once when we have a useful measurement basis (assuming ideal measurement requirements achieved) and Data trust quality, one can try to extract valuable information of this data experimental set. As we know, behind Big Data context and business intelligence needs, data is obtained directly from different kinds of sources, and variable as bigger they are. For example, many companies that uses Big Data philosophical approach in order to build a strategy obtain their sources from intern relational data basis, or social networks income, and external statistics about this company. In one hand, technological devices (software and hardware) have been improved with better data management tools. At the first century of computational data manipulation, Relational Data Bases has been a pretty good example of data recovery. Big data is not always a set of lots of data entries of random information. We think inductively, as we discovered (as mankind and scientists) that nature information presented before our perception, and then to our measurement activities, most of the times this studied phenomena presents hidden valuable structures and information about deep and fundamental properties about its very nature.

1.2 Content and Structure.

This article is structured as follows:

Sections 3 & 4 are written in order to set a formal proposal from a TDA "Invariant Method" as a regular basis against currently used methodology. We stand for the idea that there are mathematical and ontological errors among data understanding approach, and there are also mistakes made for data grouping. We also think invariant method would be a strong method due to Topological Argument.

In section 4 we describe benefits and advantages about using characteristic class analysis on regular natural phenomena, and we built its pragmatic, but also natural relation to Big Data Analysis. Characteristic class analysis is a powerful tool from topology, from where we obtain topological invariants of a physical system (data set are included as segment representations of physical real phenomenon), as long as topological approach can be set it first.

In section 5 we discuss a topological argument, which has been used over other science and technology disciplines, and it show us inductively how useful can be. We explain how topology stands to deep analysis and understanding of things based on this model, and these amazing properties came from the very complexity of topology, Finally, in section 6, we propose a qualitative description of invariant method, considering the present discussion.

1.-Introduction
2.- A Critic on Current Data Approach
3.- Big Data Structuring Information in TDA
4.- Characteristic Analysis on Data Manifolds
5.- Motivation Paradigm & Discussion
6.- Qualitative Description of Invariant Method

2 Criticism on Current Data Approach

2.1 Two Critical debates to Consider on TDA.

There are two different aspects to consider while we discuss Big Data volumes; the first one is the main approach and methodology chosen in order to fit an adequate mechanism onto real data nature. The other one, stands before the question whether data volumes might be ordered in one way or another. Both aspects are discussed in the next sections.

2.2 Understanding High Levels of Volume Information.

High Big Data levels are not only bigger numerical data sets, but it also includes variety on data nature, that is the reason for which structuring become complex inside Big Data context. Many of Big Data charts are ruled by non-relational principles, because of the complexity level and variety of information.

Since debate points presented before, it is evident, that scientific approach and point of view from basic and fundamental sciences, have not impacted already on data science. Therefore, many of visual and conceptual choices taken before, inside data base culture, have been taken ignoring transcendental reasons of information essence. We understand that data itself is not enough; i.e. although data matters, it also matters its structure, and its ontological nature, whether if its focus is holistic or reductionist. We might consider that data volumes are sets with elements, many times numbers, or observable variables, which are part of two natural phenomena:

1.- Proper understanding of data mining.

Under TDA scope, it doesn't matter, in order to build an inverse number model, the original source of data, but we do must know its mining. This means, that no matter what kind of phenomena we are trying to manipulate, TDA might work. But we also want to know about the nature of data. Although we lie on the same universe, and we presume the same laws of physics applying for almost every phenomenon -with an evident exception for those quantum process- the nature of a physical system, has a great influence in its proper understanding, specifically when we take a stand on ontological perception of systems. We are discussing this last point in next paragraph, but we might underline that, since the beginning of a data analysis, we have to keep in mind which phenomena is involved: whether if are Markovian process, or chaotic oscilator induction, or systems for which statistical sample measures might work. We also might consider if this kind of data represents greater objects or smaller; perhaps, before a computer, a data base of a Bose-Einstein elastic collisions inside a vessel might seem similar (under data base structuring and computing optics) to a *cumulae* of stars which are bigger then high temperature microscopic gas. \\ This is relevant as long as we stay with BIG DATA approach; while relational standard data bases are in some way predictable, because of the great amounts of knowledge that we got, in the other hand, Big Data has an information data entries mixed variety.

2.- Variables which represents data are always changing.

If we take a specific sample from a stochastical system, there is always the fact that, this sample would be just the "picture" and arrangement of an infinitesimal segment of time, for which the system evolve continuously. We are not talking about static data, at least not in all the cases, but this variation phenomena over observables, appears often. This last factor leads us to consider not only data volumes, but data entries elements in motion. Understanding motion as if these elements were part of a rate change of its properties with respect to time.

One can say that measured information taken for data sets our just static images of a dynamic greater phenomena. Despite of Big Data it's not the first "Dynamic data base", but also many of not relational data bases were [3], we now know, following predictions of [4] in 2020, 50 billion of devices will be connected. The whole bulk of information would increase at the time that gets dynamic, and maybe hardware technology takes a step forward in a gap. Speaking about data sources, and considering the sources itself, one can find a hierarchy of data entries inside a Big Data system, based on its relevance as we see in [5].

3 Big Data Structuring Information in TDA.

One of the problems to deal with around data science context, is the general vision of a data set presented. We must be ready to understand if given an arbitrary data set, it presents observable characteristics over Big Data information volumes. We also might be able to conclude which methodology would be the optima for research. Speaking about Big data and its structure, it presents two properties; Big Data stands for non-relational data bases, with active sources, working as a recovery information open system. In the other hand, Big Data admit vast kinds of information, and several types of data, and we should consider both aspects of it in order to apply proper methods based on adequate interpretations.

3.1 ETL Engineering and Data Setting Preparation.

Data transformations or “ETL” (Extract, Transform and Load) methodology is very common among data analysis warehousing and processing. Many of the companies with a large scale of data clusters, are asking for TI advisers to manipulate and ordain data as they ask for. Therefore, structuring information and data warehousing are needful operations inside industrial data science context. Not always, ETL operations has to do with statistical measurements and computing, but very often, both activities are complementary. ETL provides lots of strength to manipulate information high level volumes. One of its advantage in data structuring is a proper timing; this means that a good BD manipulation platform, would be able to compute thousands of operations in a short time in order to manage data for transformations or just simple queries. ETL has been the best tool until now to Big Data analysis. Although, ETL is a great tool, Big Data Analysis has not even started in its maximum reach. We are not diminishing ETL tasks, but ETL stage, is just the first of many possible stages that one can find in BD analysis process. The next step to apply mathematical analysis once data has been ordered, is to prepare data for a mathematical treatment.

3.2 Data Arrangement as mathematical object.

Data entries filling information warehouses might arrange into a mathematical object. Although, one can compare a computational data base with a mathematical matrix arrangement and it doesn't appeared any difference, at least from graphic interface sight. But there is indeed a difference, and it is a big difference. In one hand, table chart of a data base has limits, and also has defined determined aloud operations to query, transformation, extraction, counting elements, joining tables, ordenation, selection, rounding, etc... other commands. This are relational data bases commands (SQl, NoSQL, MySQL, etc), although the first data science approach to Big Data was executed by users of structured data manager software. Evolution and demanding needs of Big Data problems, has taken us in order to develop adequate software, methodology and focus approach. Speaking about the same example, a matrix has notable advantages over simple data table and charts; we know that many of these operations named above, might be done by matrix operation algorithms, although, it is not the same nature of a matrix compared with a simple data chart. A matrix itself (perhaps filled with elements of data chart or whatever other kind of elements) has singular properties inherited from linear algebra theory. Then, it is possible for a matrix to attached linear properties, speaking about it as a set of vectors, represent on a regular linear base, for which data arrangement entries are projected. These vectors generate the whole Linear Space and one can use their already known properties and theorems. One of the greatest resources from linear algebra applied to data analysis thru matrix arrangements, is the spectrum analysis, for which eigenvalue spectrum might be computed, and it is possible to obtain useful information about data source and its inner operations. Several applications might be published from eigenvalue spectrum and applications to data science, as Stock Market return information [7]. We might know that it exists an inner relation between eigenvalue spectrum of a matrix and topological objects, we can find an example of a graph linking matrix lying under a topological oriented manifold [8]. The previous step to topological manifold for data in TDA is data representation by graphs.

3.3 Current Considerations on Current TDA methodology.

As we discussed above, one of the main problems in current TDA approach is that after ETL treatment, data sets are directly and immediately related with graphs, in order to move forward and connect linking graph with graph structures. Although, this is not exactly a wrong option, it is an un-sufficient one, because matrix intervention among data-graph step, would make a huge difference in order to obtain better and complete information. This is because of the reasons exposed last section. Therefore, we introduce as an obligatory term to include matrix representation of data, before taking graph representation one.

Once we have a corresponding relation among data and graph thru matrix arrangement, one can define formally what we call as "graph data" and look out for a data science property called "data shape". Data, under this optics would be ready to be analyzed mathematically speaking by topology principles and, for our case, topological invariants.

4 Characteristic Analysis on Data Arrangements

Characteristic class analysis is a branch of topology which consists into the look for special properties of mathematical spaces, which remains equal under continuous and isomorphic transformations, among topological spaces. There are different kinds of this properties, as Euler numbers [11], Betti operators, certain kind of polynomial, matrices, and some algebraic groups as cohomology, homology and homotopy groups.

4.1 Topological Background Foundation.

Topology in general, stands for study of metric spaces E for which continuity is granted from a proper distance definition. These spaces admit continuous transformations among open sets, and they are locally compact, second numerable and count with Hausdorff space properties [11]. We say that a N-dimensional figure which can be represented algebraically or geometrically, and that counts with topological requirements, is a N-dimensional 'topological manifold'. There are several types of this manifolds. We think, based upon many former research works, that different data arrangements, which can be represented by graphs, may have different topological manifolds- Even though this method is a hard-core one, for the same reason, we must say that, it is a several numbers of different kind and dimension manifold. Thus, we might be ready, perhaps, not to know exactly the whole bulk of topological properties, but the methodology to compute these. There are well known topological properties for 3-manifolds which in topology are (4 dimensional spheres), and we know from Thurston geometrization theorem. However, there is a lot of information not recognized yet in classification theory of manifolds. This is the reason why there would be many empty knowledge vacuums when we speak about specific kinds of manifolds. An adequate and optima work in this field, it is the one with collaboration with constitutive elementary disciplines; mathematical data analysis, topology and computational data manipulation.

As we defined before, the greater operation hierarchy on topological manifolds are continuous functions on these spaces. This allow us to relate to geometrically apparent different spaces thru its shape, as topological equivalent. Formalism and mathematical theory of algebraic and combinatorial topology lies on this foundation. We say that a continuous map *f* is a homeomorphism if it is a double implicated function among two topological spaces. Topological manifolds might comply this requirement.

We are seeking these properties in data manifold. Many of T. invariants are well known nowadays, however, there would be a time when it is necessary computing a special case, based on the same very methods.

4.2 Hidden Data Information among Homeomorphism Transformations over Data Manifolds.

Due to topological argument, while we are trying to understand a given information sample of a determined system, we are not looking exactly to understand the whole amount of information (or in this case, data entries) but we are concerned about generators of the given set. If a given arbitrary set is represented by a data graph or data manifold, or even a data matrix, one can look only for those mathematical objects which generate this very object. These indicators are, in the great majority of cases, the same as topological invariants. And because these objects are generators of group, there contain hidden information about system analyzed which is not visible at a simple superficial analysis.

4. 3 T. Invariant Applicable List.

Topological invariants, are defined depending on the kind and dimension of manifold. There are different kinds of invariants and its respective computing methods for 3-manifolds then for 4-manifolds. Although, the same invariant idea may be preserved following characteristic classes which are generalizations of the same very invariants, working over N-dimensions. These work it is not done, and topologists, are always working on this general invariant classification structure. Therefore, a set of possible computable invariants is proposed in the following list:

1.- Betti Numbers [12]; For graphs, it is the number which describes de times that a cut over vertex might be done to reach a graph into a tree graph.

2.- Betti Groups; [12]; These are chains and homotopic groups.

3.- Homotopy Groups [12]; Homotopy groups are basis topological generators, these defined and gathered homology characteristics inside algebraic abelian groups. There are classified into Cohomology or Homology groups.

4.- Euler characteristic [14]; It is a relation among vertices and graph triangulation nodes and homology of its respective topological manifold. Computing method is well known for 3 and 4 manifolds. And there are some studies made for higher dimensions.

5.- Jones Polynomials [13]; These invariants, are presented as polynomials, obtained usually from knots embedded in 3-manifolds. There are some kind of graphs which represents knots as their topological manifolds. Although knots are not orientable manifolds, it is possible, most of the cases, to compute its respective Jones polynomial.

6.- Euler invariant numbers [8]; These are basic elements on continued fractions, there might be direct invariants themselves, or entries on matrix invariants.

7.- Seifert Invariants [16]: Homology 3-Spheres with Seifert invariants are filled with those. These fibers might be chosen as a convenience, and they represent characteristic invariants for chosen manifold.

There are many sub-kinds of these last invariants, but generally speaking, these are the main topological invariants, useful to analyze properties of manifolds, and for this case, data manifolds.

4.4 Invariant Computing.

Invariant computing, might be done with informatics algorithms, recurrence rules, polynomial formulation, series expansion, matrices transformation, analytic computing of algebraic homology groups, combinatorial invariants of graphs, and geometrical properties analysis. There are also formulations given for higher dimensions which can be expanded and used if it is the case.

5 Motivation Paradigm & Discussion

5.1 A Regular Argument for TDA

Topological argument stands for applications of topology over different kind of sciences. For example, algebraic topology and combinatorial one, has been used in some string theory models. There have been used also in computing [16] geometrical properties on quantum and quantum field models. They are also involved, in neural networks analysis, and informatics net architecture. The main reason for success using topology ideas as application tools over other branches, is that we think, based on transcendence of results, that topological properties represents something further than a just a higher geometry. It presents, perhaps, ontological escensce work field for natural interactions. Not only speaking about space-time structures on cosmological models, but in every cause-effect emerging phenomenon. Relations among objects, structured systems and geometrical representations, physical work frames, and mathematical observables. might be modelled by topological models. We think thou, that objective reality can be represented and written in terms of topological invariants. If we represent a given arbitrary phenomenon of physical reality with any case of structure (algebraic or geometrical) then it would be possible to analyze it since algebraic or combinatorial topology point of view, respectively.

5.2 Totality, Fragmentation and Topological Frame offered to Big Data Analysis.

Holistic ontological approach for science, stands for a unified reality in all terms of this very reality. Our perception and analysis capacity as human beings, is always trying to analyze a fragmentation of all things, in order to comprehend better what we have perceived with our senses. But this fact never can deny the fact that we are living a unified and not fragmented reality. Perceptional fragmentation of nature knowledge might be useful to study some things of our environment. But we could make severe mistakes if we don't consider all the elements of nature together. Bohm's approach [17] to this argument, invite us to consider scientific and physical frameworks where we can count on this single reality, constructing again all the fundamental pieces of a given phenomenon. Topology provides a proposal for this issue. Its proposal is based on the fact that, while in geometry, physical interactions are given among object lying on geometrical frame, in contrast, in topology, interaction among two given objects, came not on topological framework, but as topological transformations. Big Data is a warehouse and data machine for several amounts of information coming from the very outside of the BD system. Despite of data recovery seems to show information as a fragmented set, topological invariants and approach might treat it as a one single phenomenon for which its behave relies only in data shape.

5.3 Invariant Method on Data Shape and its Advantage Points.
Technically speaking, one of the strong points within our method is that, most of the times, BD set arrangements represented as graphs or matrices, will be multidimensional mathematical objects. Invariant method considers N-dimensional topological transformations over manifolds (even if those came from data graphs) and there are well known results in terms of properties and invariants for 3-manifolds and 4-manifolds. Although, in the case of higher dimension sets, there are methods in algebraic topology for which a higher dimension manifold can be decomposed thru specific topological operations as splicing or JSJ decomposition [9]. This will help us to manipulate essence on BD sets in topological work fields.

6 Invariant Method: Qualitative Algorithm Proposal

We present our proposed method elements for which algorithm stands, under our first considerations and assumptions. It is relevant to point out, that these elements not only might be present themselves in method, but it also matters order. Order of execution of this algorithm is totally relevant. One of the reasons for this fact has been exposed in section 3. But besides this argument, there are other reasons to take this exact order in the application of the method. It is necessary to consider matrix formations. Even though, if the case happened, and a data analysis process stops at data matrix stage, it is possible to find (not topological information) from data set, thru linear analysis of eigenvalue spectrum from matrix.

6.1 The elements.

6.1.1 Big Data Setting arrangement
Big Data set might count with all mathematical requirements to relate data table charts with matrix entries. The whole ETL ordaining, transformation and preparation process might be done in function of matrix arrangement needs. There might be also, an adequate mathematical algorithm lying over a proper software to get fully BD information on representation matrix.

6.1.2 Data Matrix
Once when BD entries are filled as a matrix arrangement, there must be a proper linear algebra manipulation on matrix diagonal to get canonical Jordan forms and Laplacian-Euler partial diagonalization [8] in order to set matrix blocks ready to categorical representation in a data shape graph. There are some properties which might be observable since this stage of the process, such as Euler numbers in continued fractions [8], so we can say that observable data shape begins to arise from matrix arrangement stage.

Big Data demands a higher mathematical support and environment; 3-dimensional vector matrix or tri-lineal maps, and even higher dimensional matrices. Therefore, we must stop before continuing with process, to manipulate properly (even when we already settled data in ready status at computational DB), matrix arrangement. This is about dimension reduction and projection to manipulate arrangements.

6.1.3 Graph; The shape of Data.
Data graph is a graphical model of data, it comes directly as a continuous map from matrix arrangement. It represents the second appear of property named data shape. This method work with whatever kind of graph, not only tree, but non-oriented graphs also. There are some topological properties computable from this stage, such as Betti numbers of graphs. We don't consider data shape as categorically the same of data graph. Data shape is a property of data arrangements structures. \\

Once we have well stablished data graph, we might set it ready for next stage, which consist in data graph recognition as a 'simplicial complex' and its role as a triangulation of a known or not known topological manifold, these methods belong to combinatorial topology [11]. This last stage, and data graph treatment will define the type of its attached topological manifold. The abstract and formal relation among a data arrangement and a topological or geometrical structure is called data shape.

6.1.4 Data Manifold.
Once when we relate some specific triangulation to data shape graph, we can define its topology as a manifold. There is a great advantage of knowing the manifold kind. The first one is that there are plenty of literature (articles, notes and books) where many characteristics and invariants of manifolds are well known. And this literature also contained the methodology to compute these invariants and get information back to data set reality. At his point, TDA has become just a topological algebraic problem. The greater expertise in those areas come from research on topology invariants of N-dimensional manifolds. There is also a complication with some manifolds more than others, speaking about its orientation. Not orientable topological manifolds as Moebius bands or knots are complicated to analyze in terms of characteristic classes and invariants.

6.2.1 Methodology Algorithm.

Our proposal method to use TDA invariant method approach is the following:
1.- Big Data set ready thru ETL methodology.
2.- Matrix Arrangement for BD set.
3.- Data Shape Graph Function from Data Matrix.
4.- Data graph as Triangulation of T. Manifold.
5.- Invariant Computing Over T. Data Manifold.
6.- Data Science Interpretation of T. Invariants.

This algorithm might applicable if one adjusted matrix data representation to data graph, to choose adequate T. invariants. We are working currently writing a mathematical formalism on this method, and moving forward, testing method over concrete data set examples. These results might be published in additional papers.

References

[1] Topological Data Analysis, Inverse Problems, Volume 27, Number 12, Charles Epstein, Gunnar Carlsson and Herbert Edelsbrunner.

[2] An introduction to Topological Data Analysis: fundamental, and practical aspects for data scientists, Frédéric Chazal and Bertrand Michel, Arxiv: 1710.04019.

[3] A Comparative Study of Relational and NonRelational Database Models in a Web- Based Application,Cornelia Győrödi, Robert Győrödi, Roxana Sotoc, (IJACSA) International Journal of Advanced Computer Science and Applications, Vol. 6, No. 11, 2015.

[4] IIoT Big Data Collection, Storage and, Predictive Analytics, Carl Data Solutions.

[5] Big Data: The Structure & Value of Big Data Analytics, Hak J. Kim, Structure & Value of Big Data Analytics.\\

[6] Data structure for representing a graph: combination of linked list and hash table, Maxim A. Kolosovskiy, Arxiv:ftp/arxiv/papers/0908/0908.3089.pdf.

[7] Random Matrix Approach to Correlation Matrix of Financial Data (Mexican Stock Market Case), Juan Martín Casillas González1, Antonio Alatorre Torres, Modern Economy, Vol. 6, No. 9.

[8] Algorithm for Fast Calculation of Hirzebruch-Jung Continued Fraction Expansions to Coding of Graph Manifolds, Fernando I. Becerra López, Vladimir N. Efremov, Alfonso M. Hernández Magdaleno, Article ID:59458,8 pages, 10.4236/am.2015.610149.

[9] Special positions for essential positions tori for link complements, Joan S. Braman and William M. Menasco, Pergamon, Topology Vol. 33, No 3, pp. 525 ~556, 1994.\\

[10] Characteristic Classes, John Milnor, 1974, Princeton University.

[11] Combinatorial Topology, Alexandrov, Dover Books, 1998.

[12] Algebraic Topology, Allen Hatcher.

[13] Witten, Edward. Quantum field theory and the Jones polynomial. Comm. Math. Phys. 121 (1989), no. 3, 351--399. https://projecteuclid.org/euclid.cmp/1104178138.

[14] (2;m;n)-groups withEuler characteristic equal to2asb, Nick Gill, the electronic journal of combinatorics20(3)(2013), #P81.

[15] A mini-course on topological strings, Marcel Vonk, Arxiv; hep-th/0504147.

[16] Quantum invariants of Seifert 3–manifolds, and their asymptotic expansion, Søren Kold Hansen, Toshie Takata, Geometry & Topology Monographs, Volume 4: Invariants of knots and 3-manifolds (Kyoto 2001)
Pages 69–87, ISSN 1464-8997 (on line) 1464-8989 (printed).

[17] Wholeness, and the Implicated order, David Bohm, First published 1980 by Routledge & Kegan Paul, ISBN 0-203-99515-5 Master e-book ISBN.